

Scale-related artefacts on (satellite) rainfall estimation

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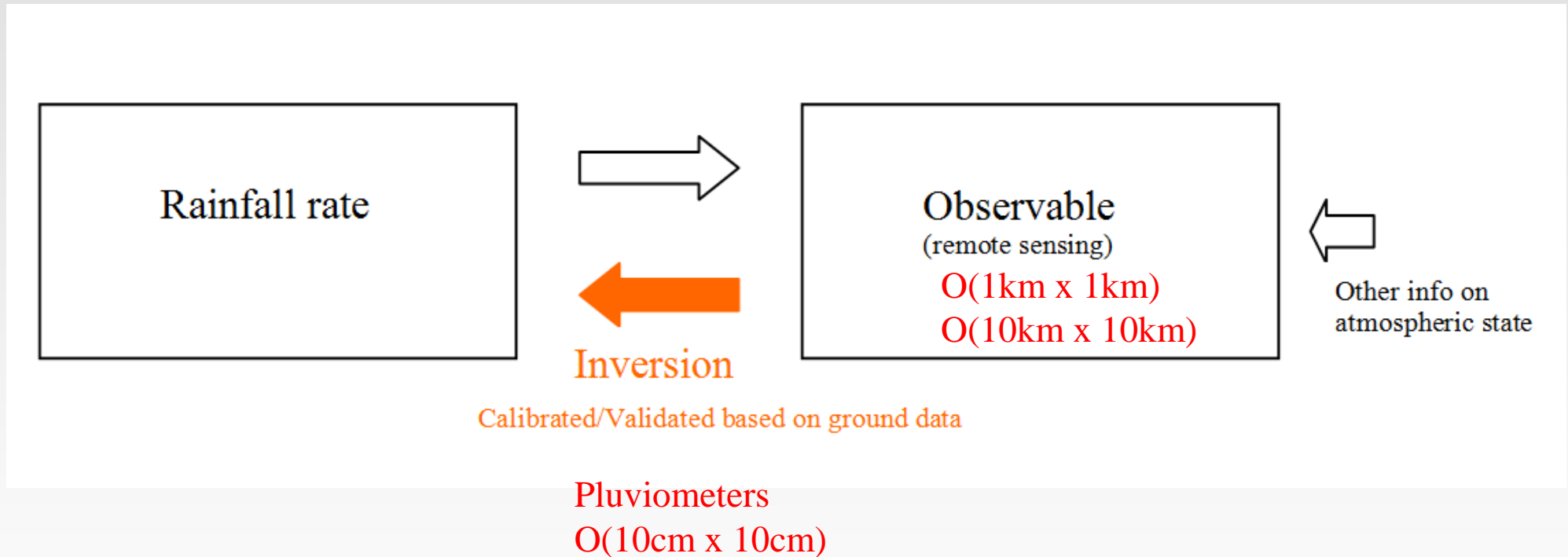
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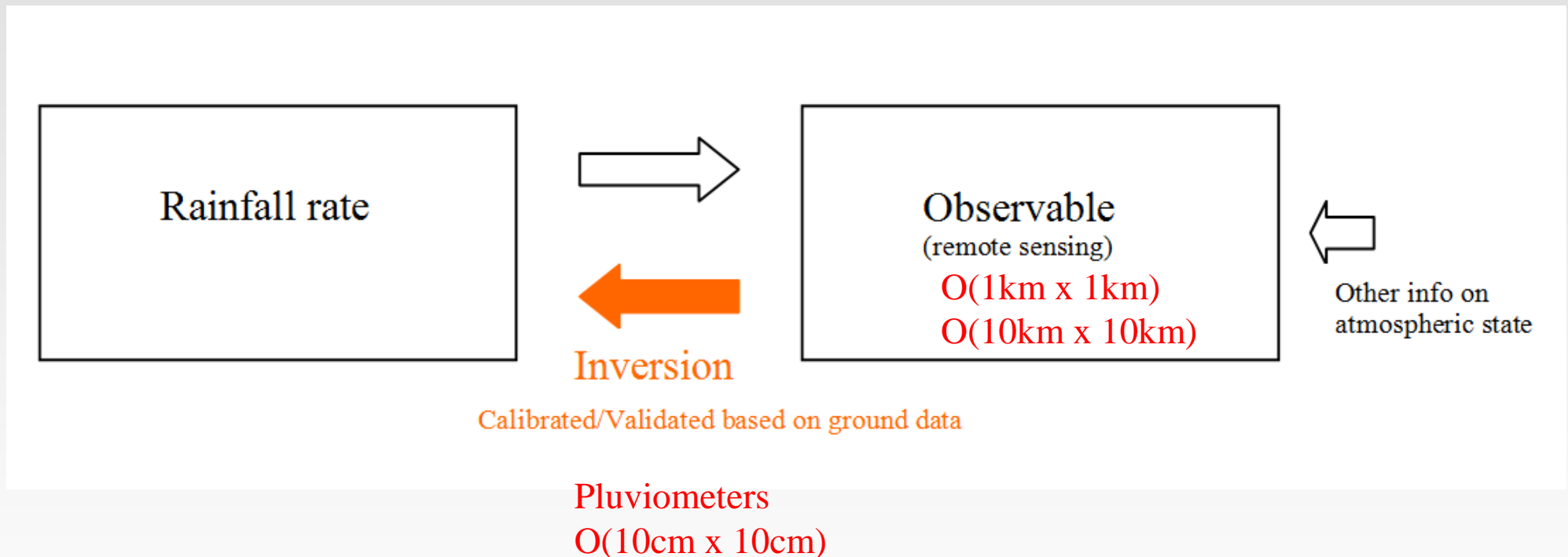


Remote-sensing and measurement scale





Remote-sensing and measurement scale



How to deal with data with very different resolutions?
Are (implicit) sub-pixels smoothness assumptions acceptable?

Following:

Example in radar precipitation estimation: Z-R relationships



Z-R power-laws

- Radar reflectivities Z (mm⁶/m³) are converted into rainfall rates R (mm/h) using parameterized power-laws, e.g.

$$Z = aR^b \quad (a,b) = \text{semi-empirical parameters}$$

- **Well-known factors affecting the parametrization**
 - Rainfall type (convective, stratiform)/Microphysics
 - Errors of conversion of altitude data into ground based values (VPR...)
 - Regression methodology (linear vs non-linear, choice of the explicative variable)
- **What about regularity hypotheses?**
 - DSDs, R and Z are considered scale-independent, smooth, regular...

Rainfall is not smooth, but scaling!

- Huge variability on a wide range of time/ space scales
- Strong empirical evidence of scaling properties for rain-related fields:

- Spectral analysis
- Multifractal analysis

Moment scaling function

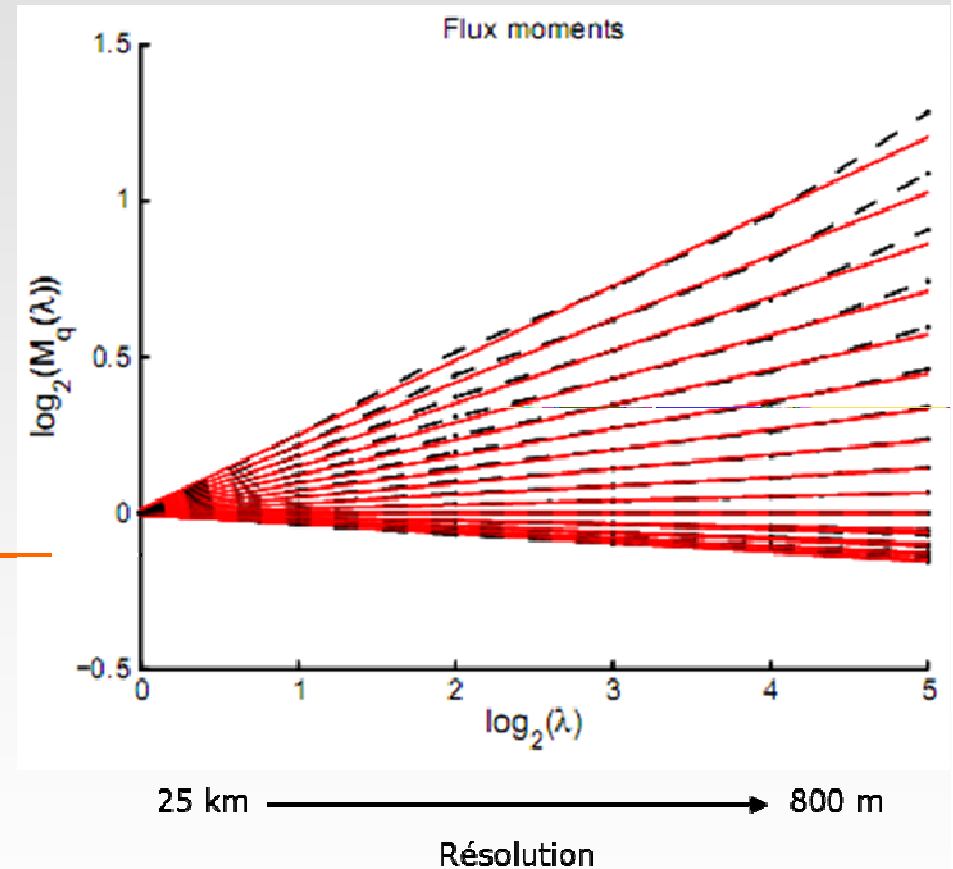
$$\langle \Phi_\lambda^q \rangle \approx \lambda^{K(q)}$$

$$\lambda = \frac{L_0}{l}$$

Resolution factor in the space domain
 l = space scale

$$\lambda = \frac{T_0}{\tau}$$

Resolution factor in the time domain
 τ = time scale



Scaling properties of statistical moments of rainfall intensities derived from RONSARD radar measurements (AMMA campaign)



Z-R revisited with multifractals

- Z and R are multifractal: $\langle Z_\lambda^q \rangle \approx \lambda^{K_Z(q)} \langle Z_\lambda \rangle^q$

- Write a Z-R power-law: $\langle Z_\lambda^q \rangle = a^q \langle R_\lambda^{bq} \rangle$

- Conseq 1: Relationship between moments scaling functions

$$K_Z(q) = K_R(bq) - qK_R(b)$$

- Conseq 2: Coef. a varies as power-law of λ :

Experimental setup

- Disdrometer data collected in Palaiseau (France)
- High signal to noise ratio (\neq radar data)

- Estimates of Z (reflectivity factor) and R at 15 s:

$$R(15s) \propto \int N(D)v(D)D^3 dD \quad Z(15s) \propto \int N(D)D^6 dD$$

- Z and R simultaneously aggregated at coarser scales: 30 s, 1 min, ..., 64 min

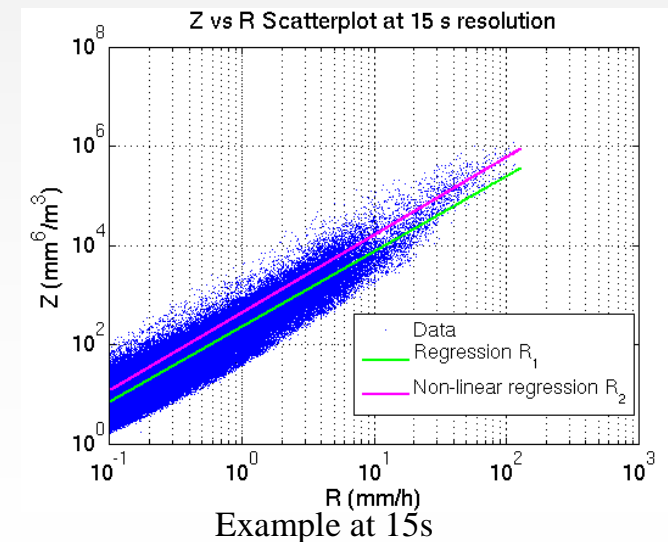
- Estimation of $a(\tau)$ and $b(\tau)$ as a function of aggregation time scale

Purpose: check if

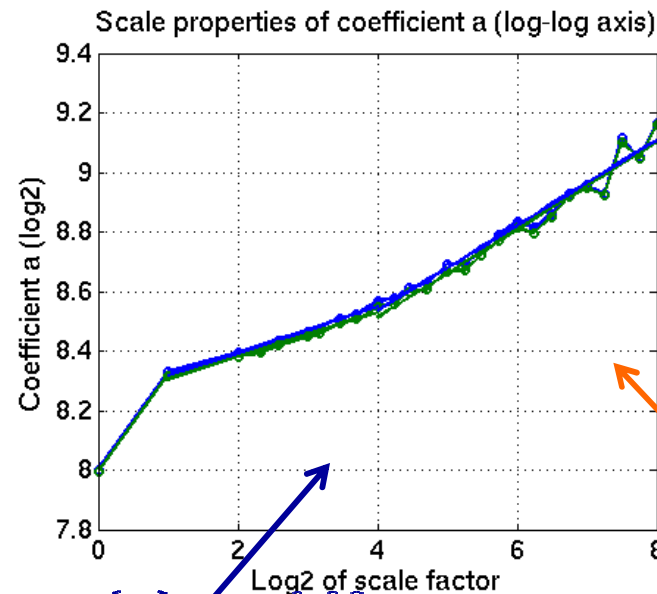
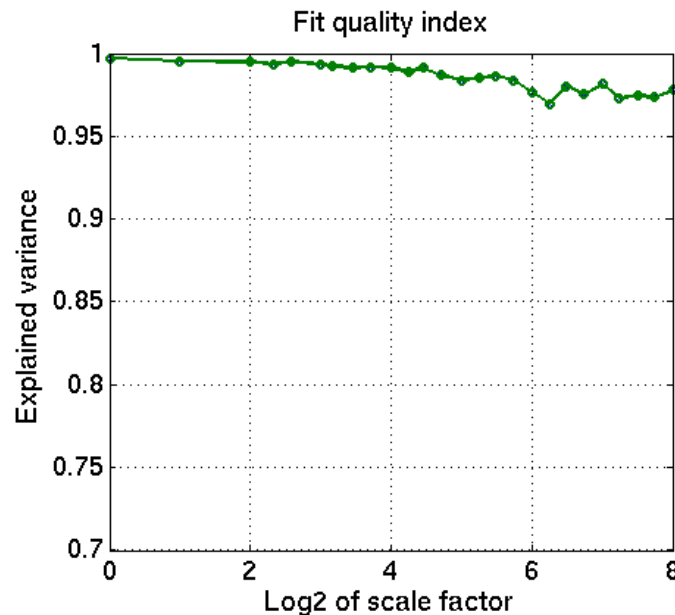
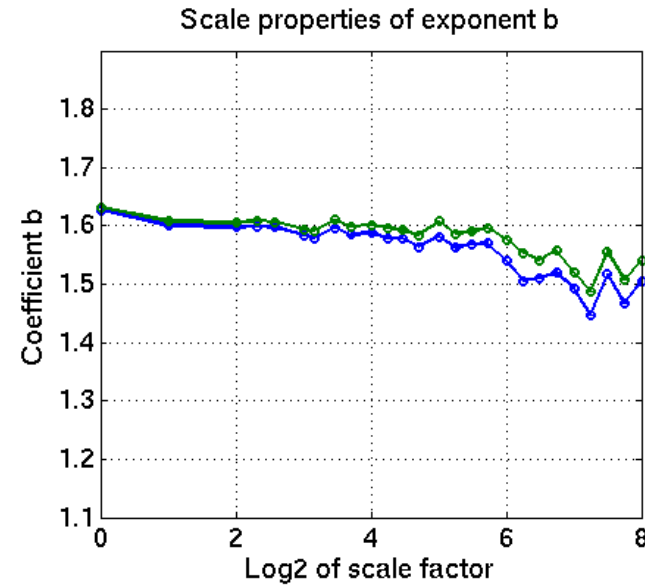
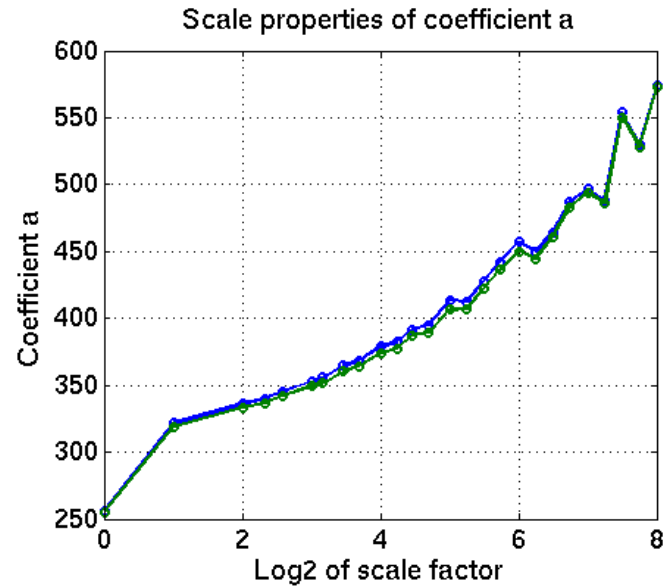
$$a(\tau) \propto \tau^{+K_R(b)}$$

?

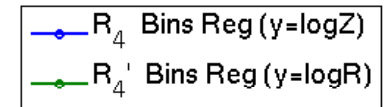
$$b(\tau) = const$$



Scale-dependency of (a,b)



$Z = a R^b$ fit:
(Log-)Linear regressions
Binned-data



XScale vs Time Scale:

- x=0: 15 s
- x=2: 1 min
- x=4: 4 min
- x=6: 16 min
- x=8: 64 min

$a(\tau) \sim \tau^{0.08}$

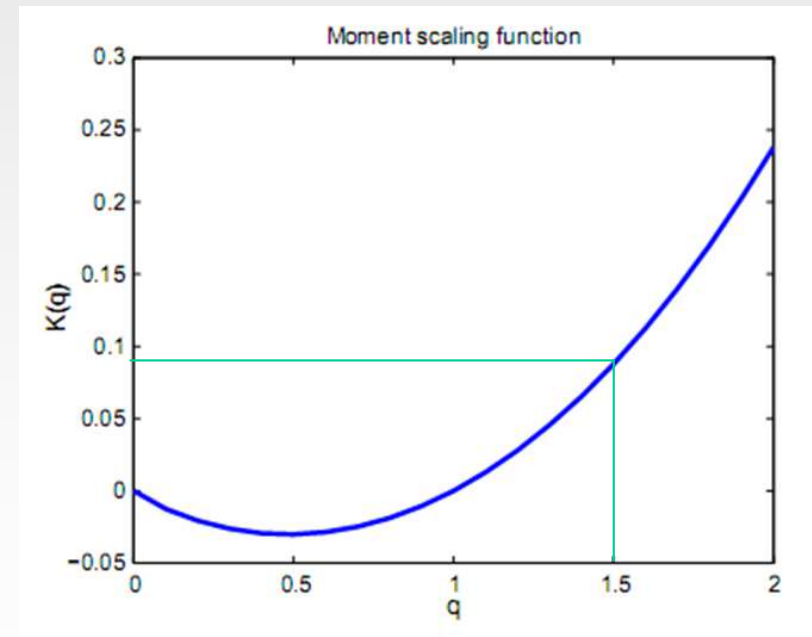
$a(\tau) \sim \tau^{0.14}$

Interpretation of the scaling law

- Empirical results confirm the scaling law of factor a in time:
 - $K(b) = 0.05 - 0.2$
 - Dependent on event type and regression methodology
 - Literature → rainfall is multifractal in the space domain also
- What impact on rainfall estimates (radar meteorology)?

$$a(\tau) \propto \tau^{+K_R(b)}$$

Empirical $K(q)$ function for the RONSARD radar dataset, scale range 0.4-25 km



$$a(\lambda) \propto \lambda^{-K_R(b)} \propto l^{+K_R(b)}$$



Interpretation of the scaling law

- Conseq: when estimating R from Z with (a,b) correctly calibrated at a smaller-scale
- ...a positive bias should be obtained (in average) on estimated intensities

$$R_{L,estim} / R_{L,true} = K(b)/b$$

- Example: if (a,b) are correctly calibrated for a ground-based radar with 1km x 1km pixels and are applied with no change for a space-borne radar with 5km x 5km pixels:
 - With $b \sim 1.5$ and $K(b) = 0.1 \rightarrow +10\%$ of relative error
 - With $b \sim 1.5$ and $K(b) = 0.2 \rightarrow +20\%$ of relative error
- More critical: the error should be larger when calibrating radars with disdrometers/pluviometers (10 cm but larger time resolution)



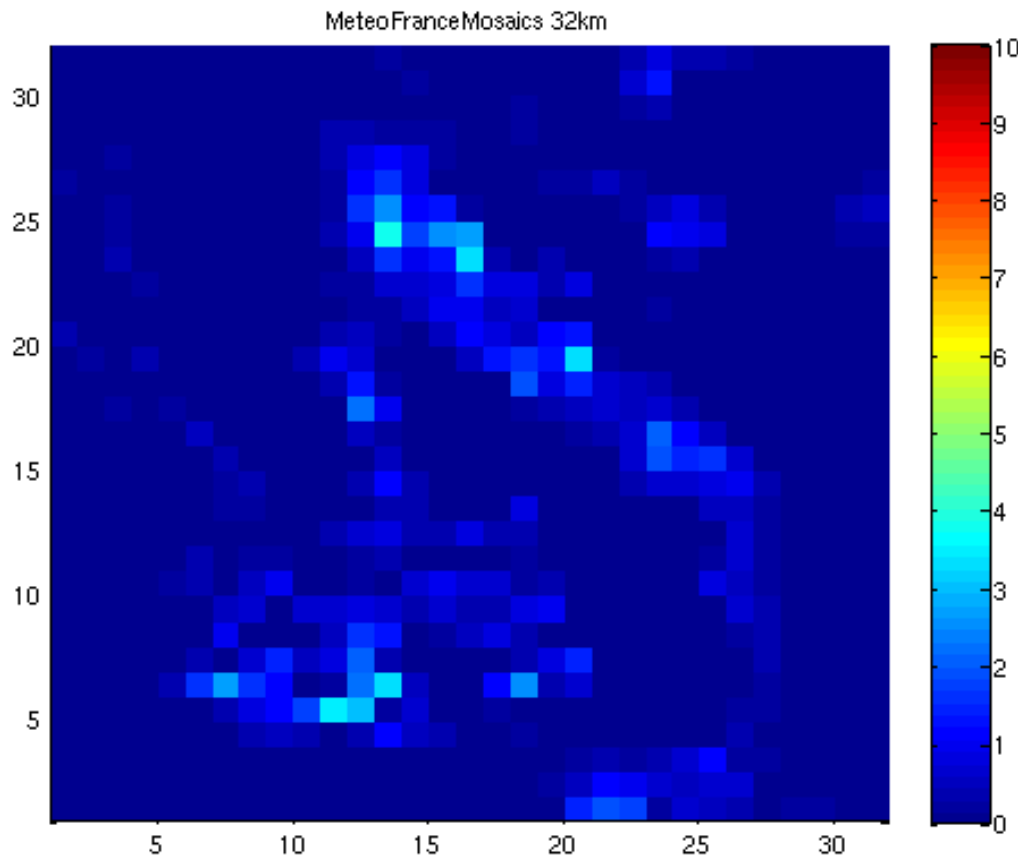
Perspectives for other sensors

- Z-R relationships are much more simple than inversion methods of satellite passive remote sensing...
 - Satellite data pixels are very large → same scale problems when comparing with gauges
 - Non-linear inversion methods should be strongly affected by scale-related errors
 - Contrary to Z-R laws, no analytical solution...
- Alternative: using multifractal simulators for inferring sub-pixel rainfall variability

Inferring sub-pixel variability

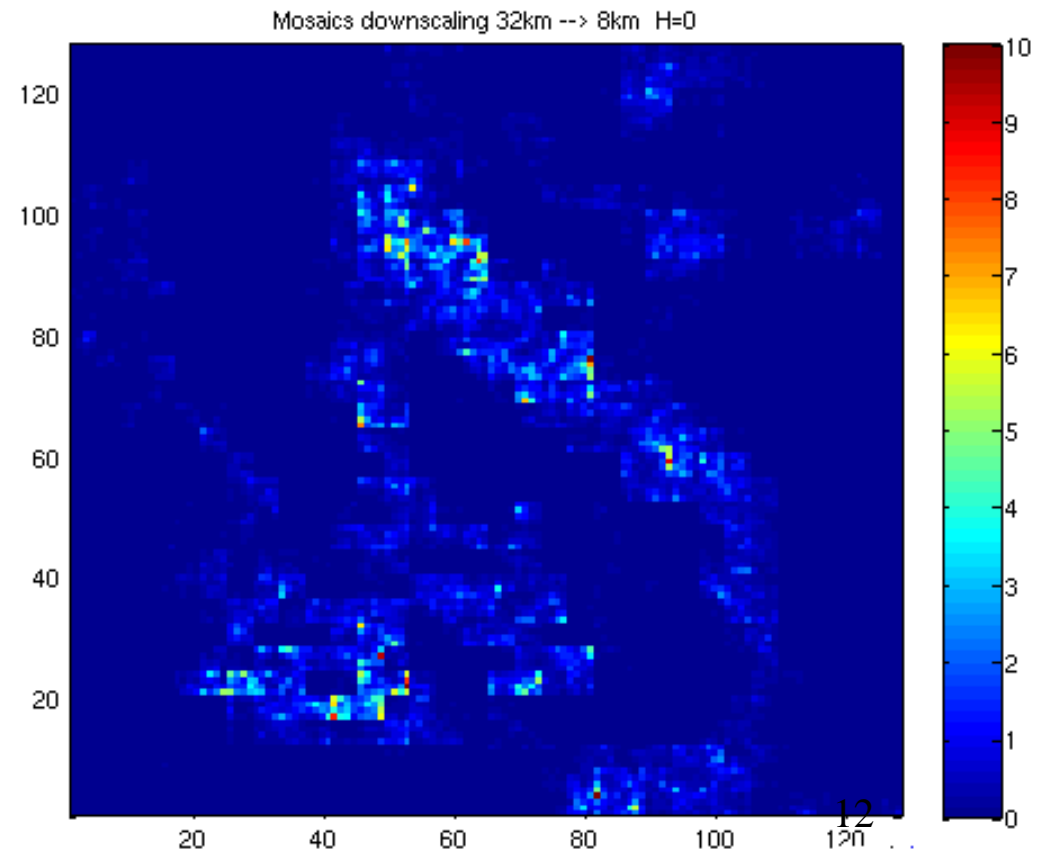
- Possibility to simulate stochastic sub-pixel variability by extrapolating multifractal scaling laws...
- By construction, accurate retrieval of the CDF at multiple scales

Rainfall composite data aggregated
at 32 km scale



MeteoFrance Mosaics rain rate (26/03/2008)

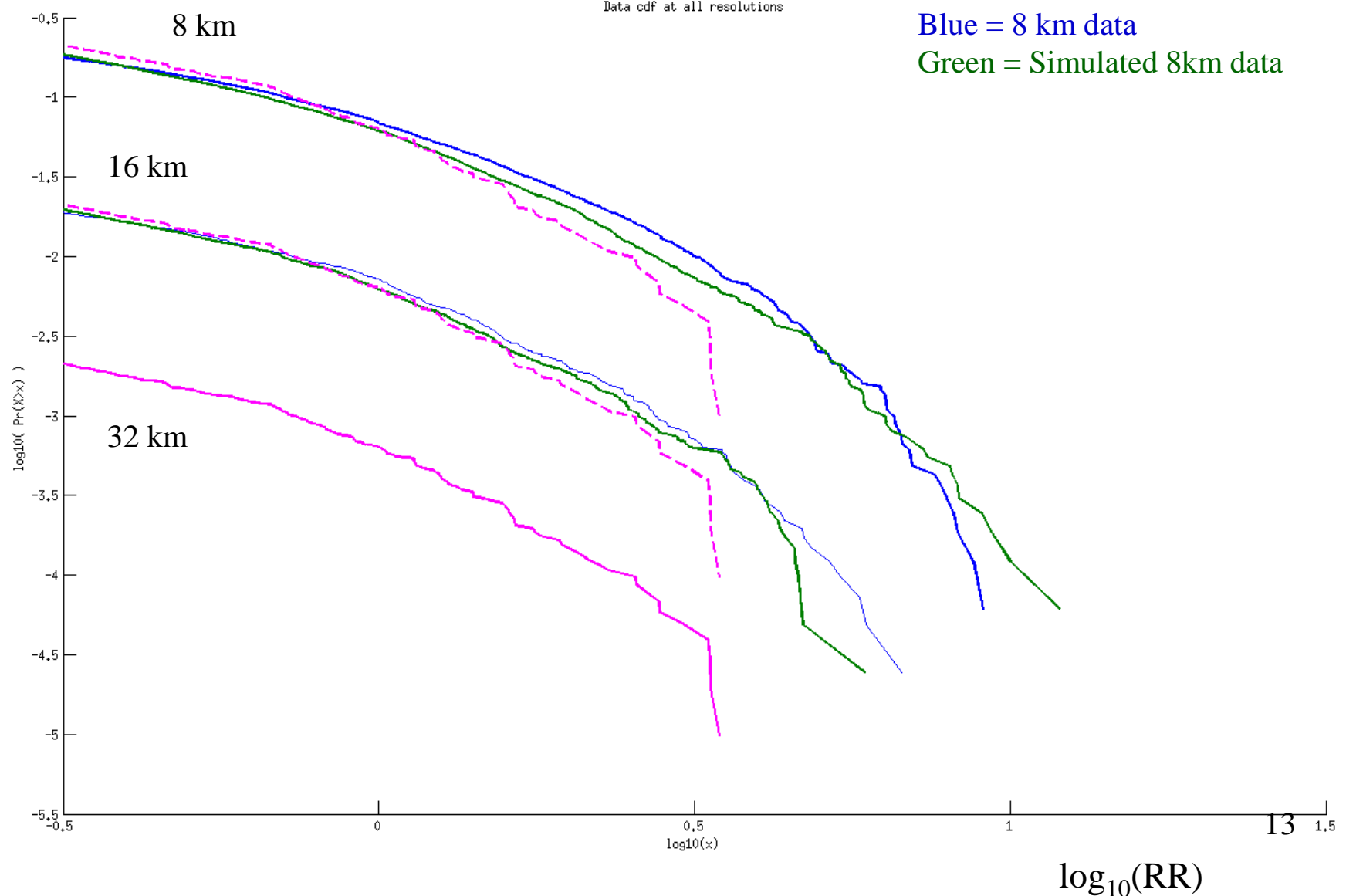
Rainfall composite data
disaggregated at 8 km scale





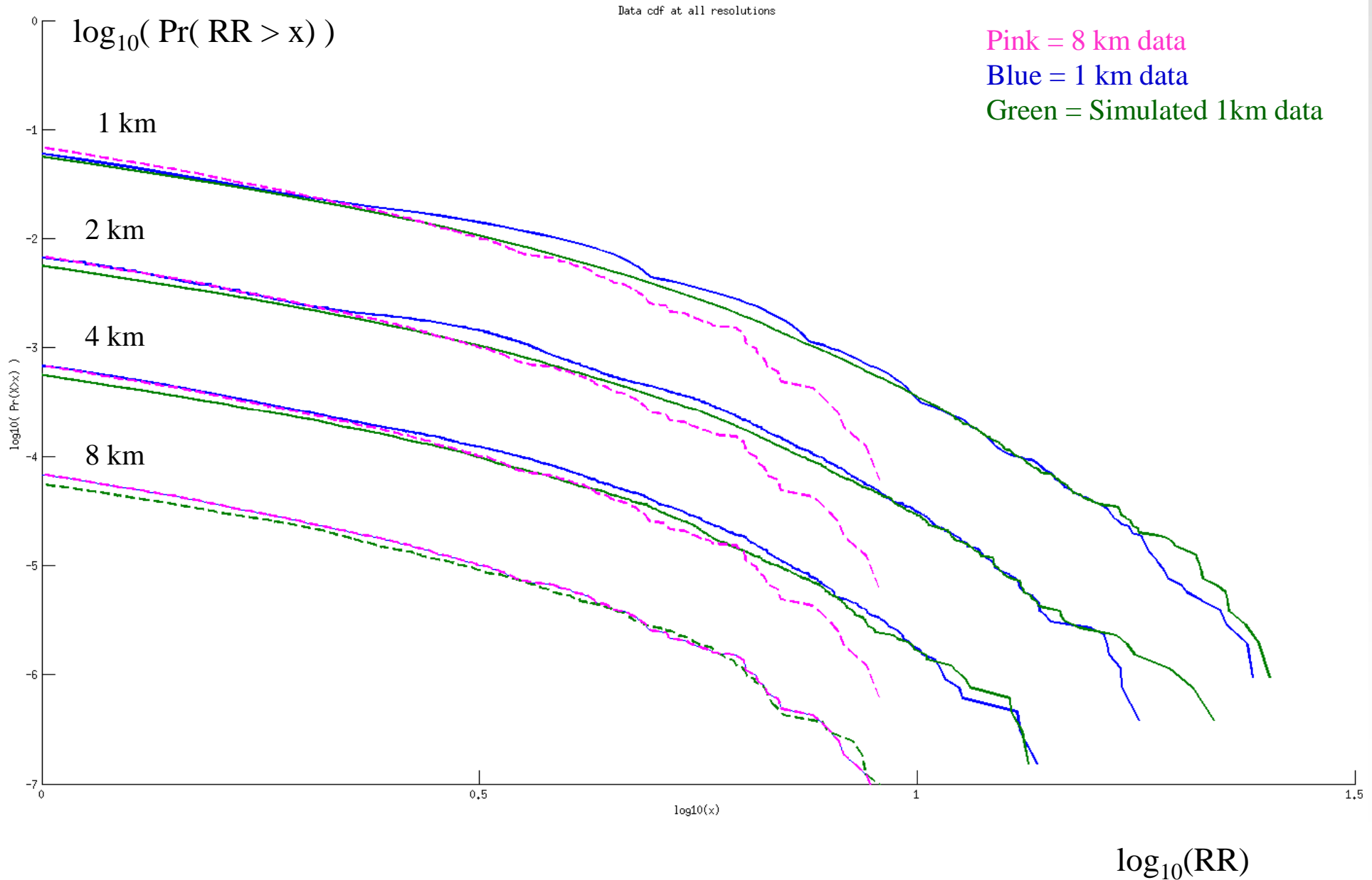
Multifractal downscaling 32-8 km (example on 1 map)

$\log_{10}(\Pr(RR > x))$





Multifractal downscaling 8-1 km (example on 1 map)





Conclusion

- Calibration and validation procedures merging rainfall data of different resolutions **are significantly scale-sensitive**
- Reason: great variability characterized by multifractal statistics
- **Multifractal formalism**: tool to understand such artifacts
 - Z-R case: analytical solution
 - Other sensors: possibility to use multifractal simulators
- **Multifractal downscaling = promising tool for rainfall data comparison and fusion**

More info on Z-R scale-dependency

Theoretical and empirical scale dependency of Z-R relationships: Evidence, impacts, and correction

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[1] Estimation of rainfall intensities from radar measurements relies to a large extent on power-laws relationships between rain rates R and radar reflectivities Z , i.e., $Z = a * R^b$. This relationship is generally applied unawarely of the scale, which is questionable since the nonlinearity of this relation could lead to undesirable discrepancies when combined with scale aggregation. Since the parameters (a,b) are expectedly related with drop size distribution (DSD) properties, they are often derived at disdrometer scale, not at radar scale, which could lead to errors at the latter. We propose to investigate the statistical behavior of Z - R relationships across scales both on theoretical and empirical sides. Theoretically, it is shown that claimed multifractal properties of rainfall processes could constrain the parameters (a,b) such that the exponent b would be scale independent but the prefactor a would be growing as a (slow) power law of time or space scale. In the empirical part (which may be read independently of theoretical considerations), high-resolution disdrometer (Dual-Beam

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Multifractal formalism (recalls)

- Power-law spectra

$$E(k) \approx k^{-\beta} \quad \beta = 1-K(2) (< 1)$$

- Statistical moments = power-law of the resolution

$$\langle \Phi_{\lambda}^q \rangle \approx \lambda^{K(q)}$$

q = order of statistics

$K(q)$ = moment scaling function

λ = resolution (here, = 1/scale)

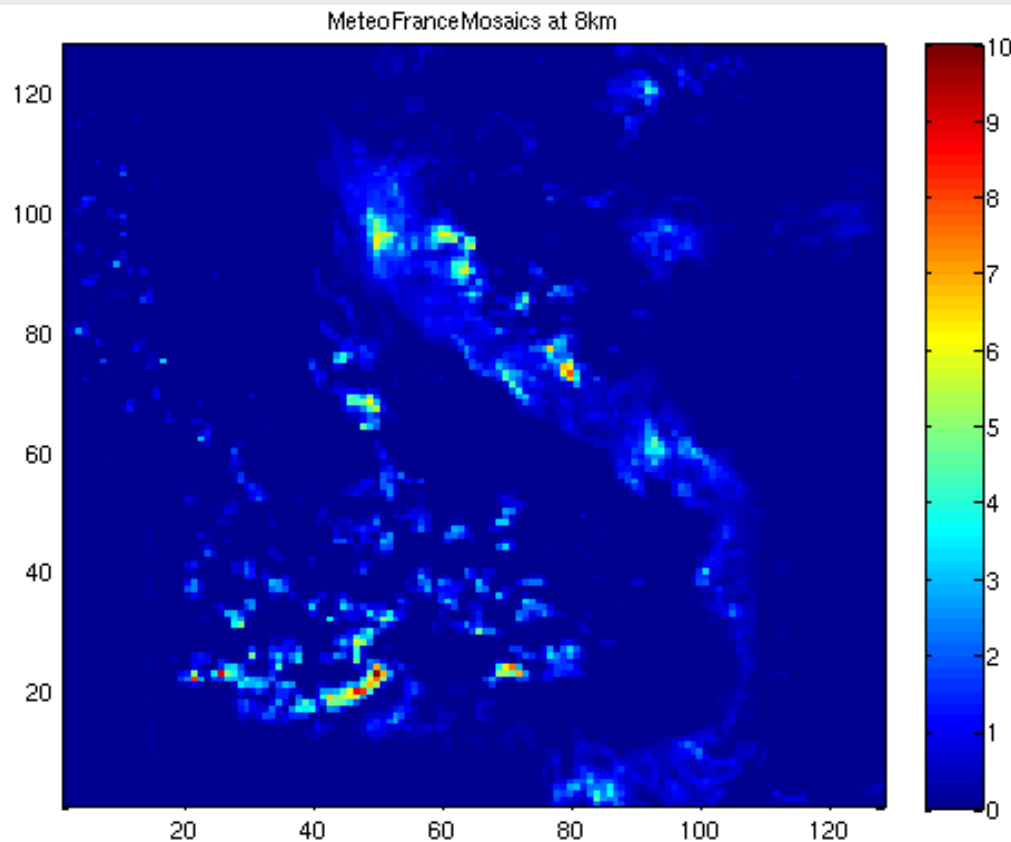
- Cascade phenomenology, with non – conservative variants ($P(q) = K(q) - qH$)
- Powers of multifractal fields are multifractal (*Tessier et al. 1993*)



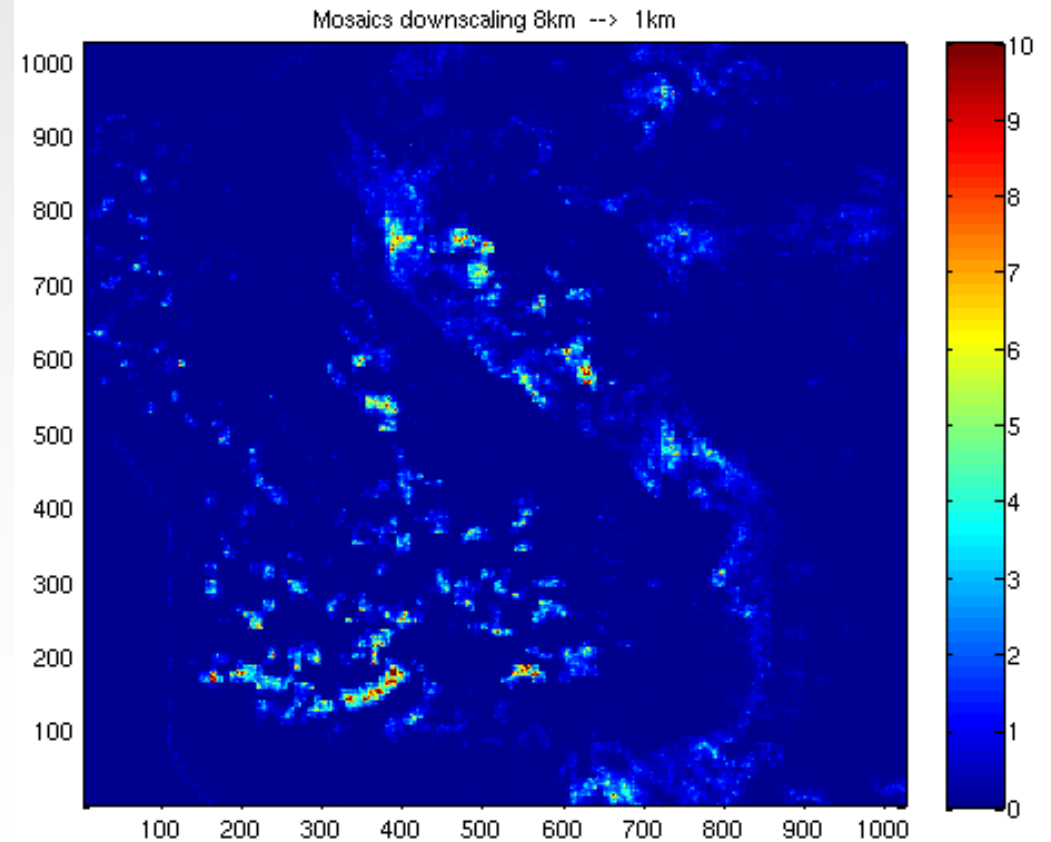
Inferring sub-pixel variability

- Possibility to simulate stochastic sub-pixel variability by extrapolating multifractal scaling laws...
- By construction, accurate retrieval of the CDF at multiple scales

Rainfall composite data aggregated
at 8 km scale



Rainfall composite data
disaggregated at 1 km scale



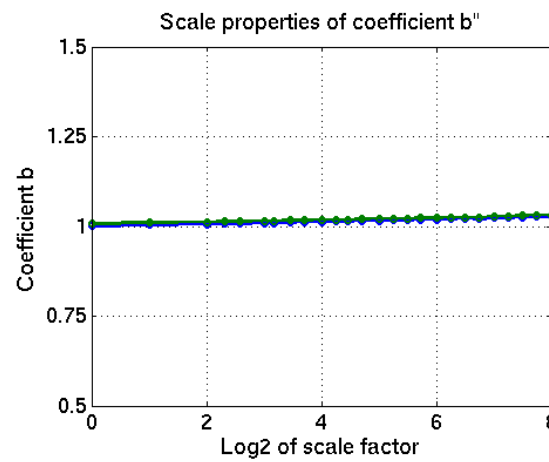
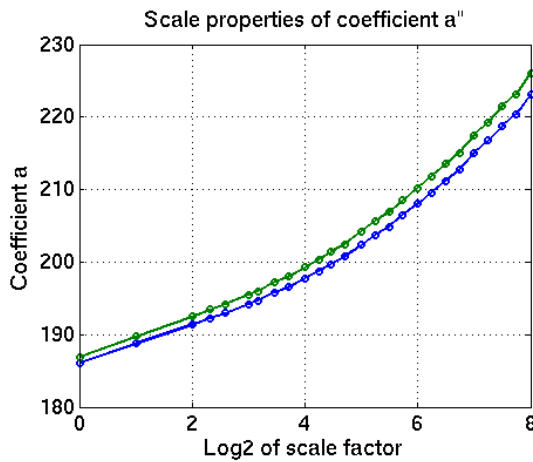
MeteoFrance Mosaics rain rate (26/03/2008)

Scale behaviour of normalized Z-R laws

- (Multiscale) Normalized Z-R- N_0^* - D_m relationships are much more stable with scale with exponents 0.02 – 0.05

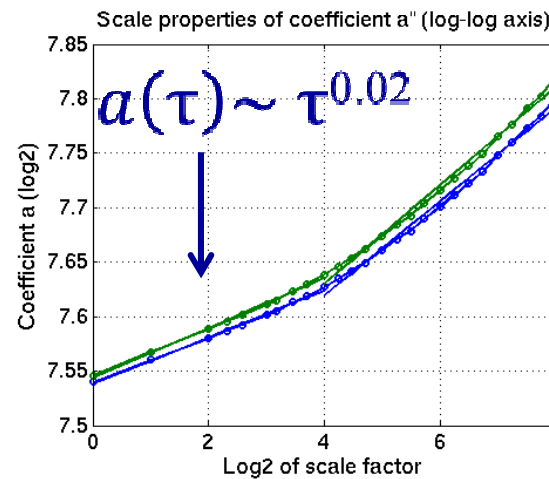
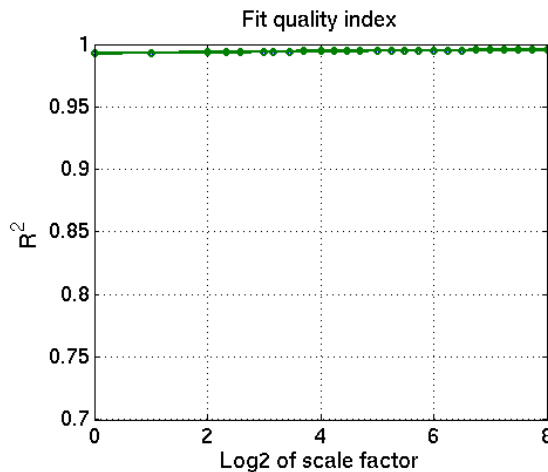
$$\left(\frac{Z}{N_0^*}\right) = a' \left(\frac{R}{N_0^*}\right)^{1.5}$$

$$Z = a'' R^{1.0} D_m^{2.33}$$



$Z = a'' R^{b''} D_m^{2.33}$ fit:
Linear/pca regressions on log-data

—●— R_6 Lin Reg on logs ($x = \log(R \cdot D_m^{2.33})$)
—●— R_6' Lin Reg on logs ($y = \log(R \cdot D_m^{2.33})$)



XScale vs Time Scale:

x=0: 15 s
x=2: 1 min
x=4: 4 min
x=6: 16 min
x=8: 64 min



Scale behaviour of normalized Z-R laws

- Testud et al. (2001) defined DSD normalization parameters N_0^* and D_m (defined from 3rd and 4th DSD moments) such that $N(D) = N_0^* f(D/D_m)$:

$$N_0^* = \frac{4^4 M_3^5}{6 M_4^4}$$

$$D_m = \frac{M_4}{M_3}$$

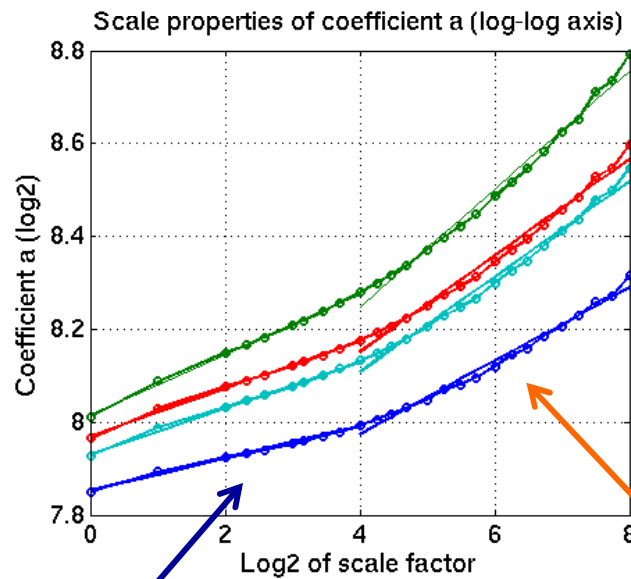
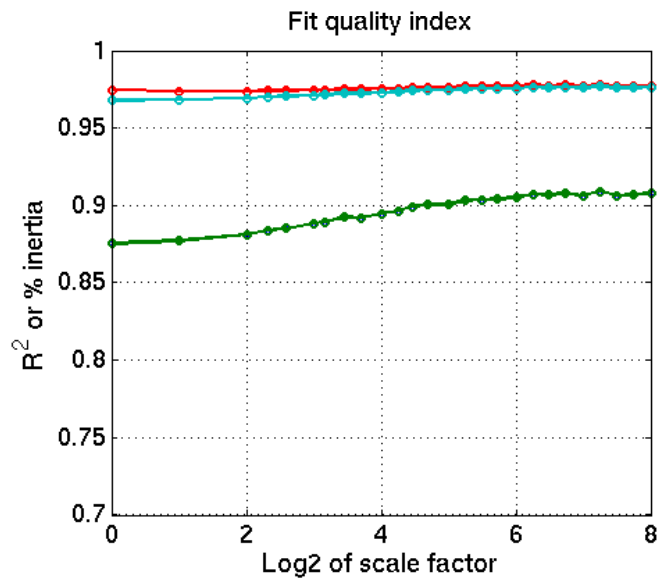
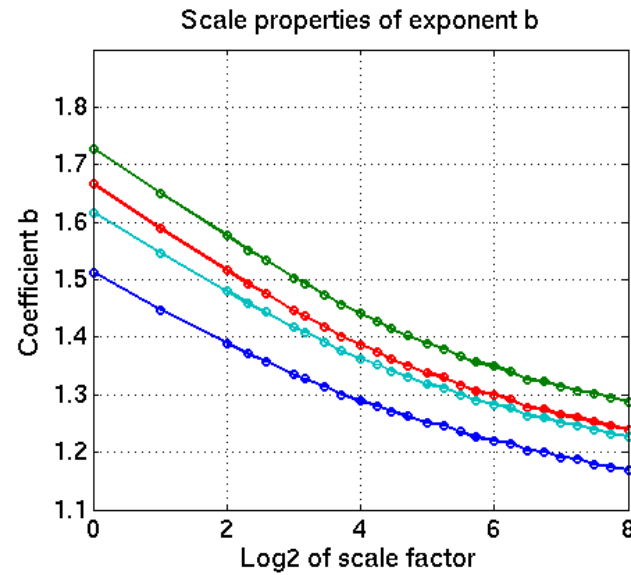
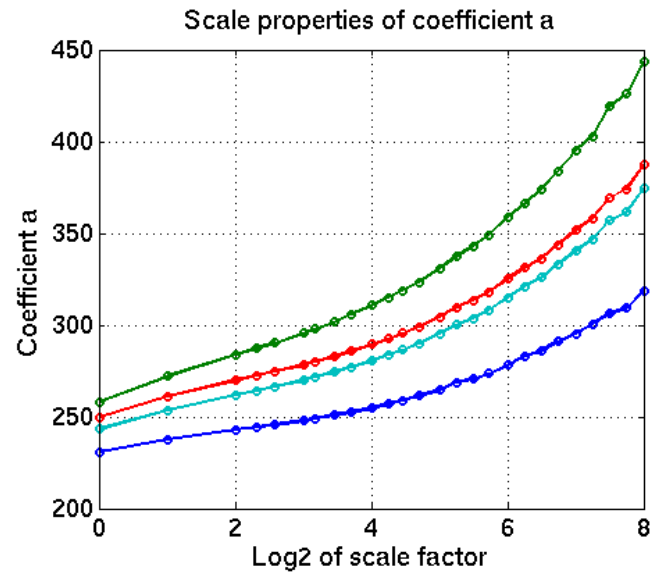
- **Modified Z-R relationships**

$$\left(\frac{Z}{N_0^*} \right) = a' \left(\frac{R}{N_0^*} \right)^{1.5}$$

$$Z = a'' R^{1.0} D_m^{2,33}$$

- Contrary to Testud et al., **we estimated N_0^* and D_m at multiple scales** from time aggregations of M_3 and M_4 of the DSD
- Then, same experimental methodology on the disdrometer data...

Scale-dependency of a and b



$Z = a R^b$ fit:
Linear/pca regressions on log-data

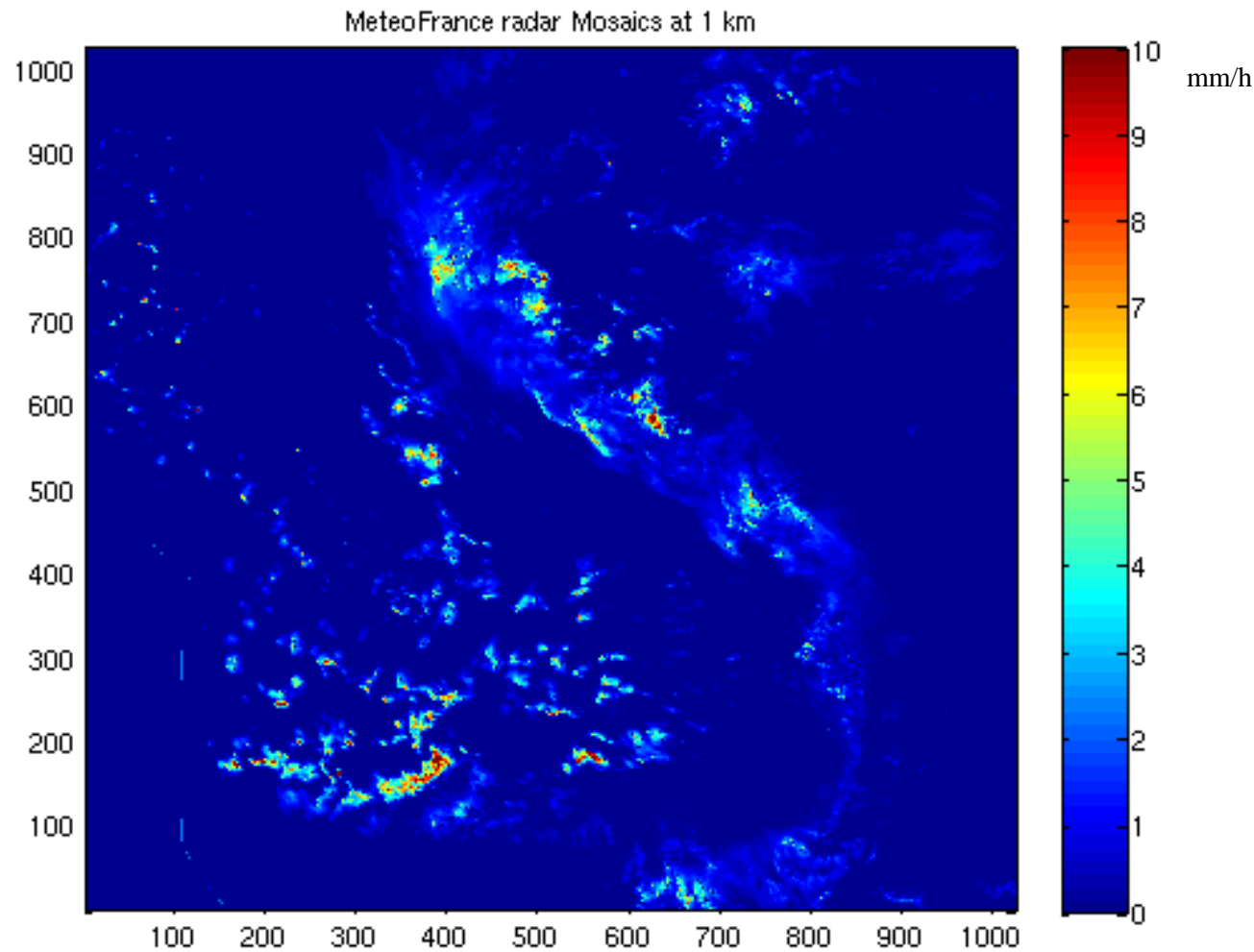
- R_1 Lin Reg on logs ($y = \log Z$)
- R_1' Lin Reg on logs ($y = \log R$)
- R_2 Acp Reg on logs
- R_2^* Norm. Acp Reg on logs

XScale vs Time Scale:
 $x=0$: 15 s
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 $x=8$: 64 min

$a(\tau) \sim \tau^{0.05}$

$a(\tau) \sim \tau^{0.10}$

Rainfall is not smooth, but scaling!



Rainfall radar composite over France (26/03/2008)