Scale-related artefacts on (satellite) rainfall estimation

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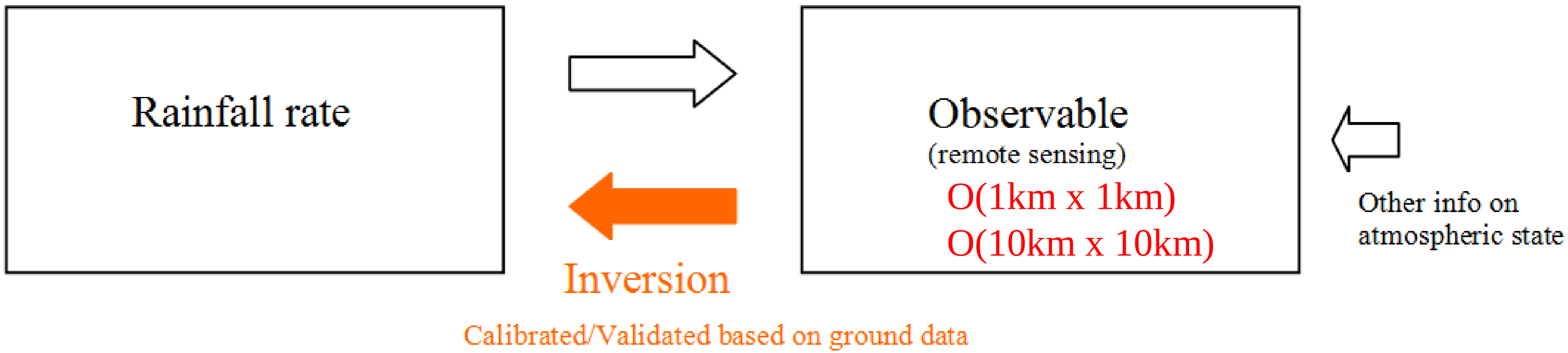
Remote-sensing and measurement scale



Pluviometers
O(10cm x 10cm)

Remote-sensing and measurement scale

Rainfall rate

Observable
(remote sensing)
O(1km x 1km)
O(10km x 10km)

Other info on
atmospheric state

Inversion

Calibrated/Validated based on ground data

Pluviometers
O(10cm x 10cm)

How to deal with data with very different resolutions?
Are (implicit) sub-pixels smoothness assumptions acceptable?

Following:
Example in radar precipitation estimation: Z-R relationships

Z-R power-laws

- Radar reflectivities Z (mm6/m3) are converted into rainfall rates R (mm/h) using parameterized power-laws, e.g.

$$\boxed{Z = aR^b}$$

(a,b) = semi-empirical parameters

- Well-known factors affecting the parametrization
  - Rainfall type (convective, stratiform)/Microphysics
  - Errors of conversion of altitude data into ground based values (VPR…)
  - Regression methodology (linear vs non-linear, choice of the explicative variable)

- What about regularity hypotheses?
  - DSDs, R and Z are considered scale-independent, smooth, regular…

Rainfall is not smooth, but scaling!

- Huge variability on a wide range of time/ space scales

- Strong empirical evidence of scaling properties for rain-related fields:
  - Spectral analysis
  - Multifractal analysis

Moment scaling function

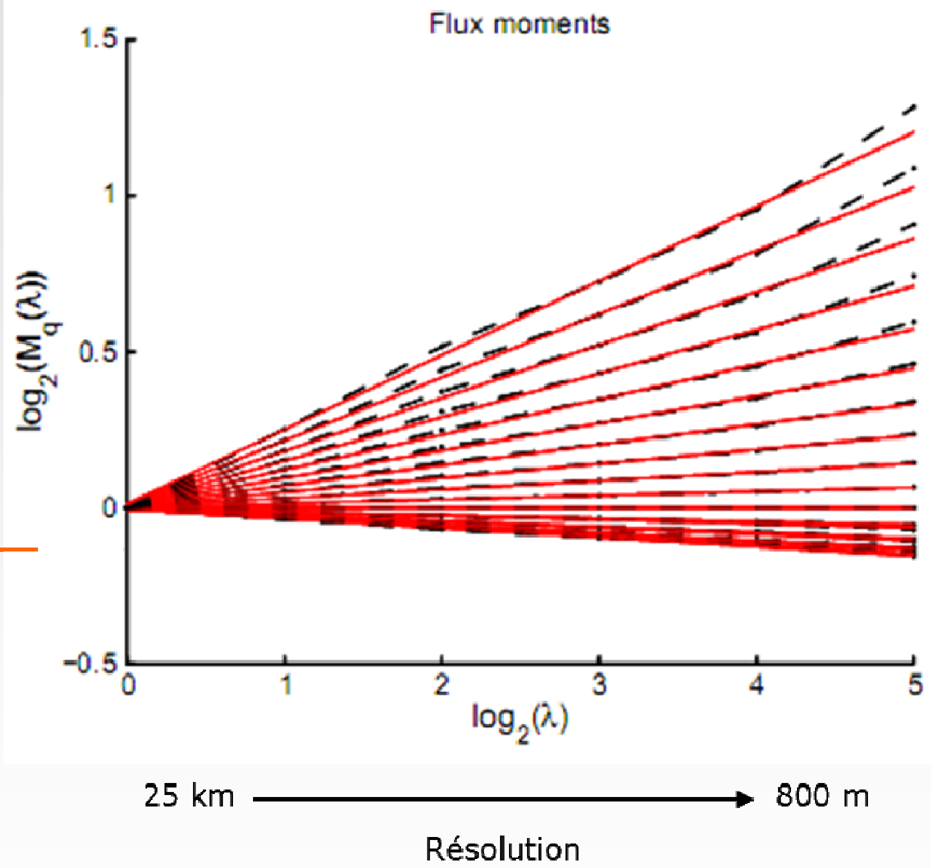
$$\boxed{\left\langle \Phi_{\lambda}{}^{q} \right\rangle \approx \lambda^{K(q)}}$$

$$\lambda = \frac{L_0}{l}$$

Resolution factor in the space domain
l = space scale

$$\lambda = \frac{T_0}{\tau}$$

Resolution factor in the time domain
τ = time scale



Scaling properties of statistical moments of rainfall intensities derived from RONSARD radar measurements (AMMA campaign)

Z-R revisited with multifractals

- Z and R are multifractal: $\langle Z_\lambda{}^q \rangle \approx \lambda^{K_Z(q)} \langle Z_\lambda \rangle^q$

  $\langle R_\lambda{}^q \rangle \approx \lambda^{K_Z(q)} \langle R_\lambda \rangle^q$

- Write a Z-R power-law: $\left\langle Z_\lambda^q \right\rangle = a^q \left\langle R_\lambda^{bq} \right\rangle$

- Conseq 1: Relationship between moments scaling functions

$$K_Z(q) = K_R(bq) - qK_R(b)$$

- Conseq 2: Coef. *a* varies as power-law of λ:

$$a(\lambda) \propto \lambda^{-K_R(b)} \propto l^{+K_R(b)}$$

For time scales:
$a(\tau) \propto \tau^{+K_R(b)}$

Due to mathematical constraints on K(q), we get K(b) > 0 for the usual case b > 1

Experimental setup

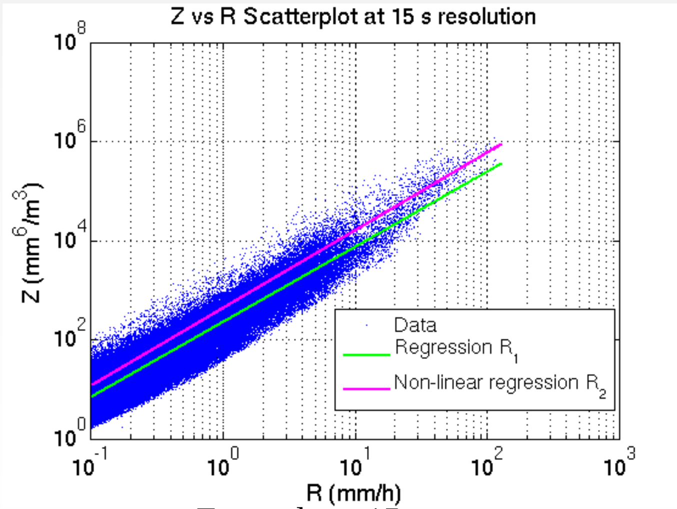
- Disdrometer data collected in Palaiseau (France)
- High signal to noise ratio (≠ radar data)

- Estimates of Z (reflectivity factor) and R at 15 s:

$$R(15s) \propto \int N(D)v(D)D^3 dD \qquad Z(15s) \propto \int N(D)D^6 dD$$

- Z and R simultaneously aggregated at coarser scales: 30 s, 1 min, …, 64 min

- Estimation of a(τ) and b(τ) as a function of aggregation time scale



Example at 15s

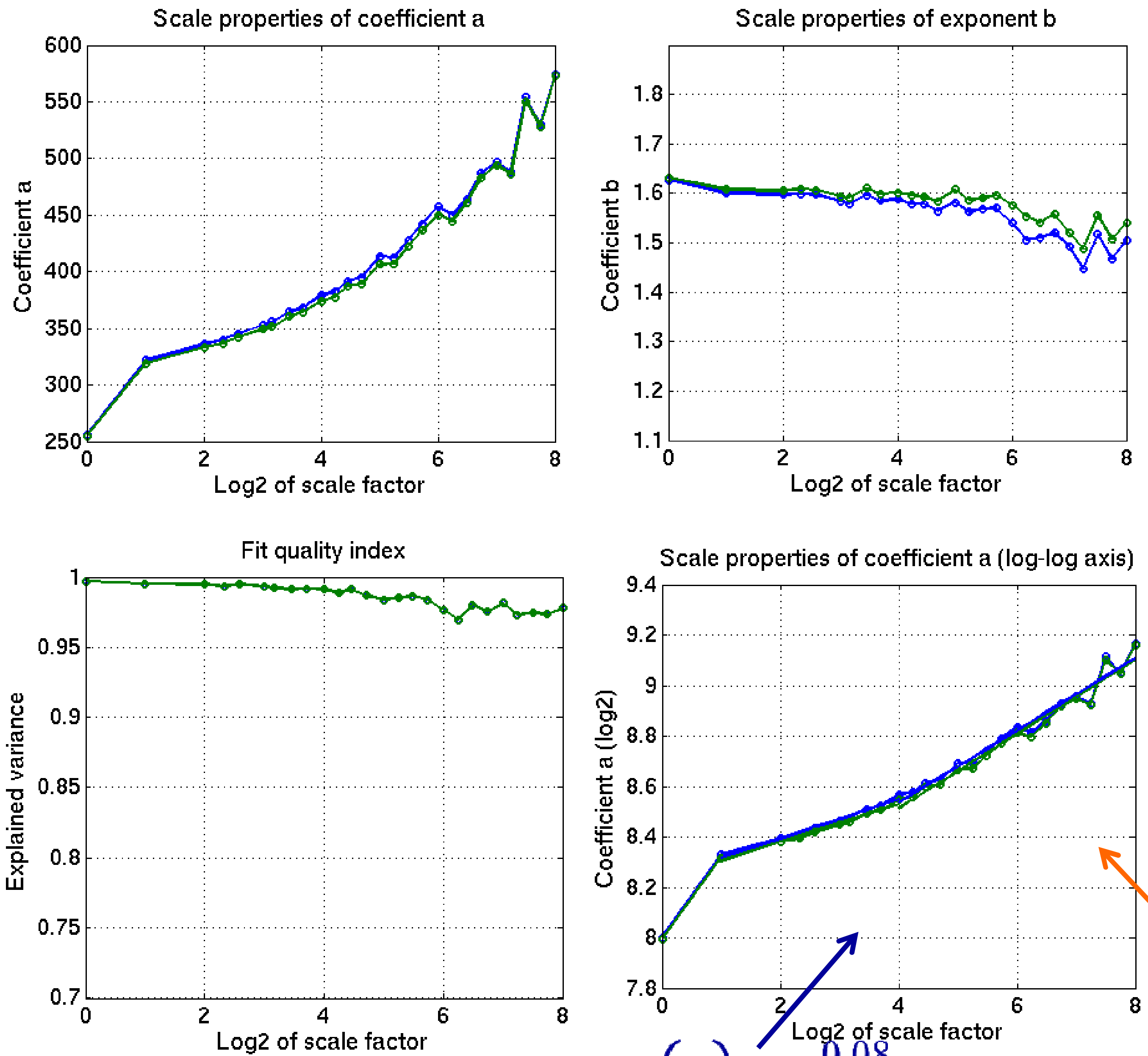
Purpose: check if

$$\boxed{a(\tau) \propto \tau^{+K_R(b)}}$$

$$\boxed{b(\tau) = const}$$

?

Scale-dependency of (a,b)



$Z = a R^b$ fit:
(Log-)Linear regressions
Binned-data

XScale vs Time Scale:
x=0: 15 s
x=2: 1 min
x=4: 4 min
x=6: 16 min
x=8: 64 min

$a(\tau) \sim \tau^{0.08}$

$a(\tau) \sim \tau^{0.14}$

Interpretation of the scaling law

- Empirical results confirm the scaling law of factor *a* in time:

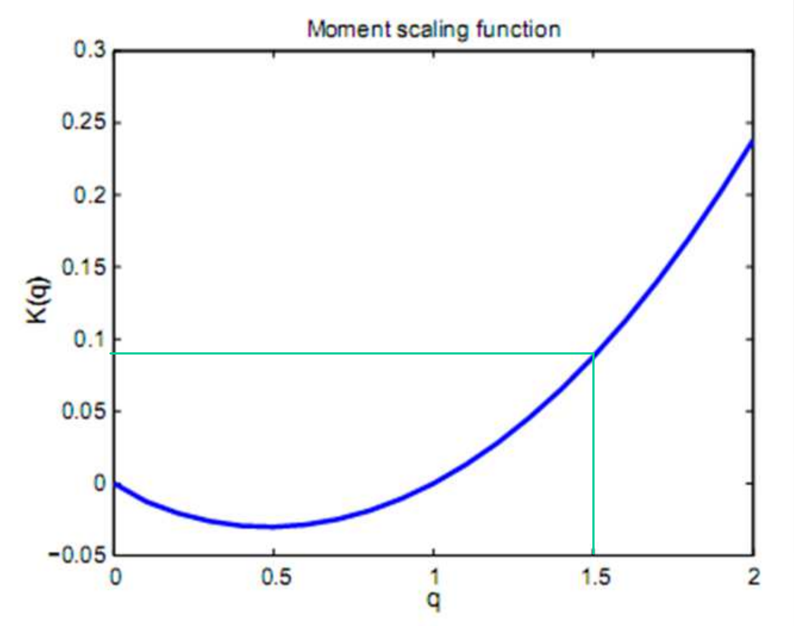
$$a(\tau) \propto \tau^{+K_R(b)}$$

  - K(b) = 0.05 – 0.2
  - Dependent on event type and regression methodology

  - Literature → rainfall is multifarctal in the space domain also

- What impact on rainfall estimates (radar meteorology)?

Empirical $K(q)$ function for the RONSARD radar dataset, scale range 0.4-25 km



$$a(\lambda) \propto \lambda^{-K_R(b)} \propto l^{+K_R(b)}$$

Interpretation of the scaling law

- Conseq: when estimating R from Z with (a,b) correctly calibrated at a smaller-scale

- …a positive bias should be obtained (in average) on estimated intensities

$$R_{l,estim} / R_{l,true} \sim (scale\ factor)^{K(b)/b}$$

- Example: if (a,b) are correctly calibrated for a ground-based radar with 1km x 1km pixels and are applied with no change for a space-borne radar with 5km x 5km pixels:
  - With b ~ 1.5 and K(b) = 0.1 → +10% of relative error
  - With b ~ 1.5 and K(b) = 0.2 → +20% of relative error

- More critical: the error should be larger when calibrating radars with disdrometers/pluviometers (10 cm but larger time resolution)

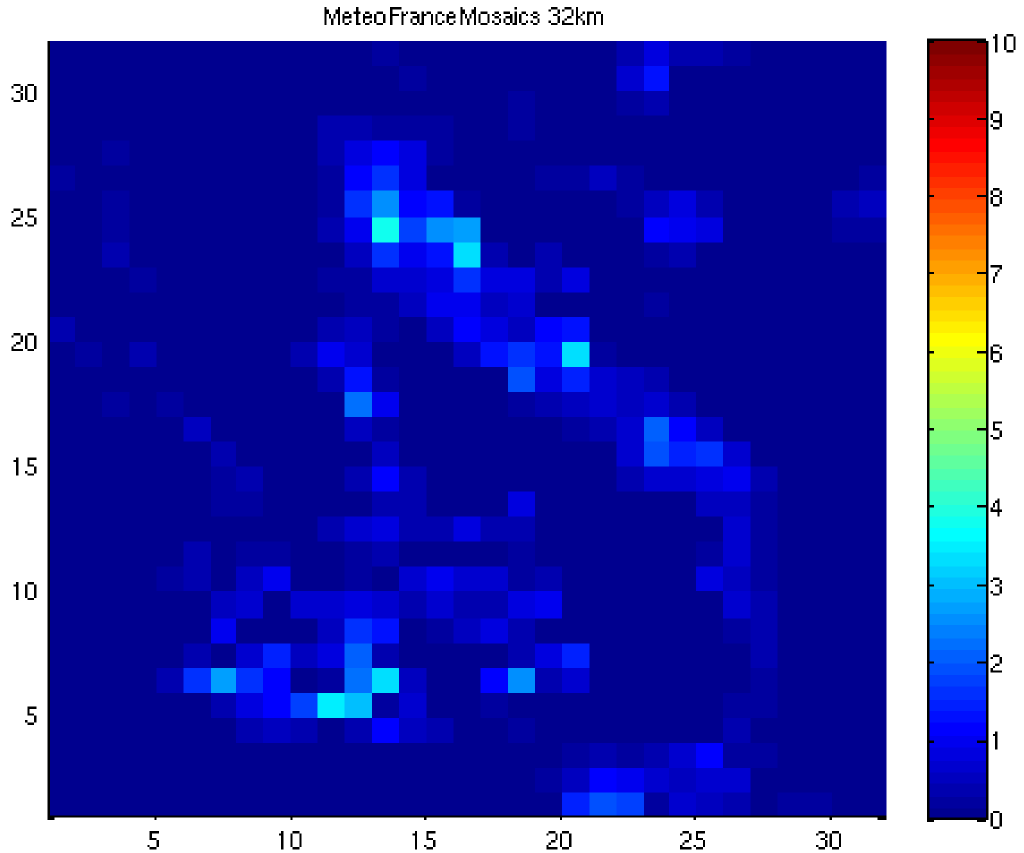
Perspectives for other sensors

- Z-R relationships are much more simple than inversion methods of satellite passive remote sensing…
- Satellite data pixels are very large → same scale problems when comparing with gauges
- Non-linear inversion methods should be strongly affected by scale-related errors
- Contrary to Z-R laws, no analytical solution…
- Alternative: using multifractal simulators for inferring sub-pixel rainfall variability

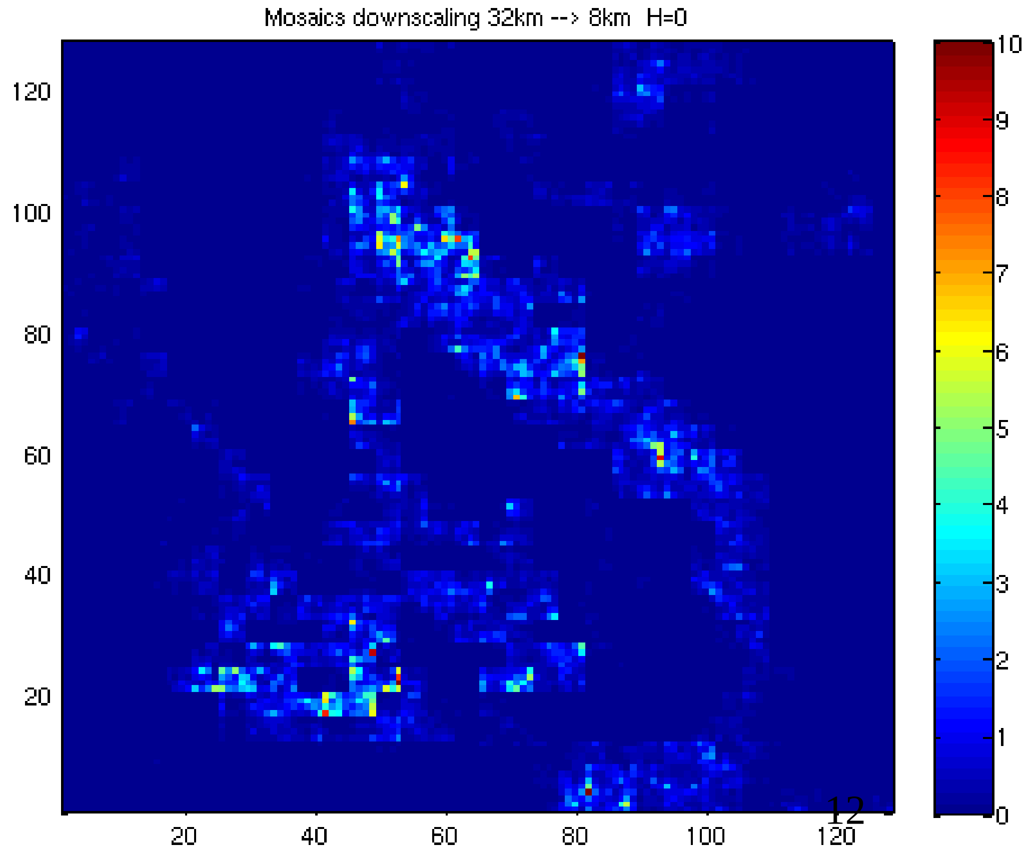
Inferring sub-pixel variability

- Possibility to simulate stochastic sub-pixel variability by extrapolating multifractal scaling laws…
- By construction, accurate retrievement of the CDF at multiple scales

Rainfall composite data aggregated at 32 km scale

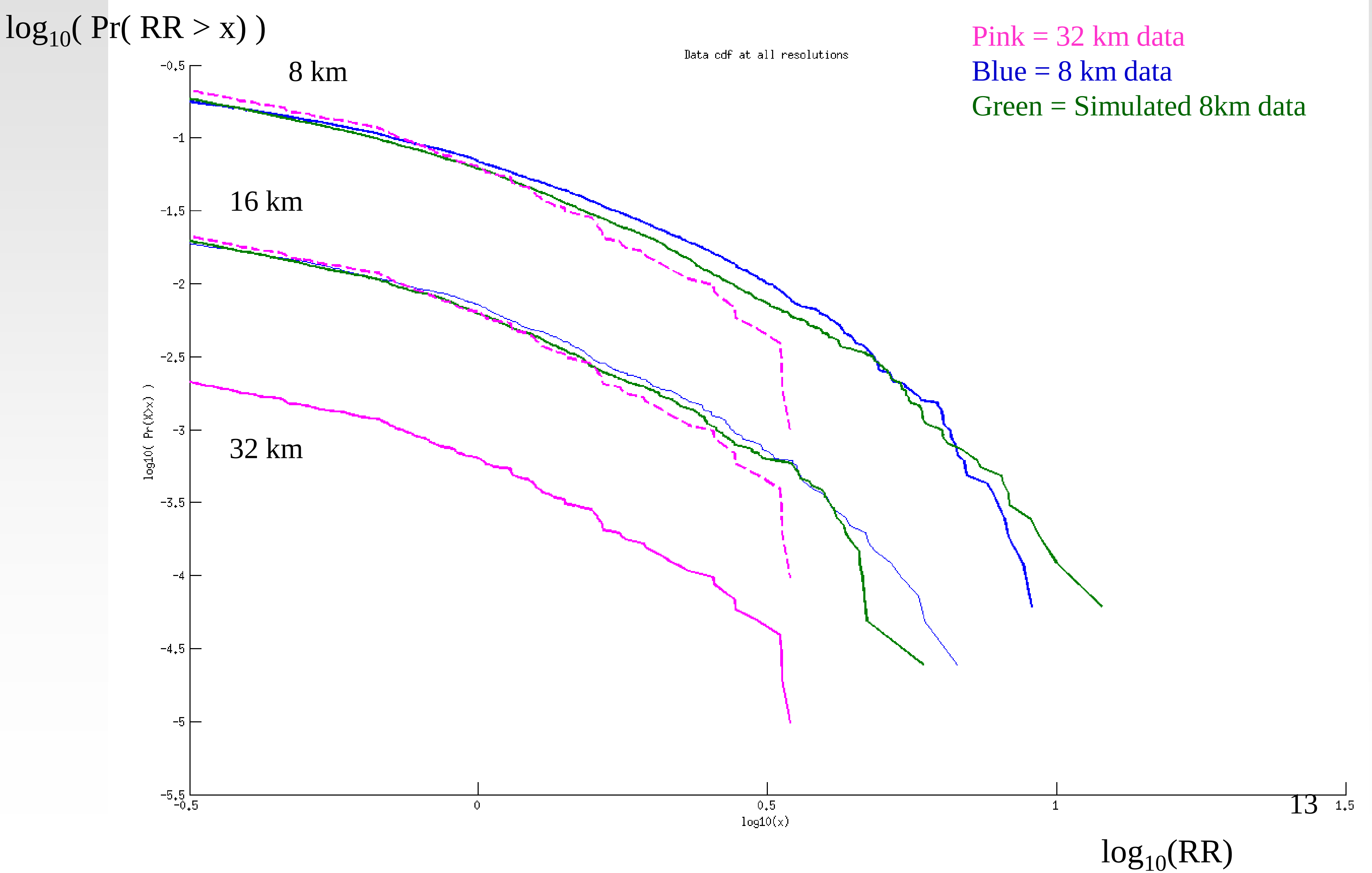


Rainfall composite data disaggregated at 8 km scale

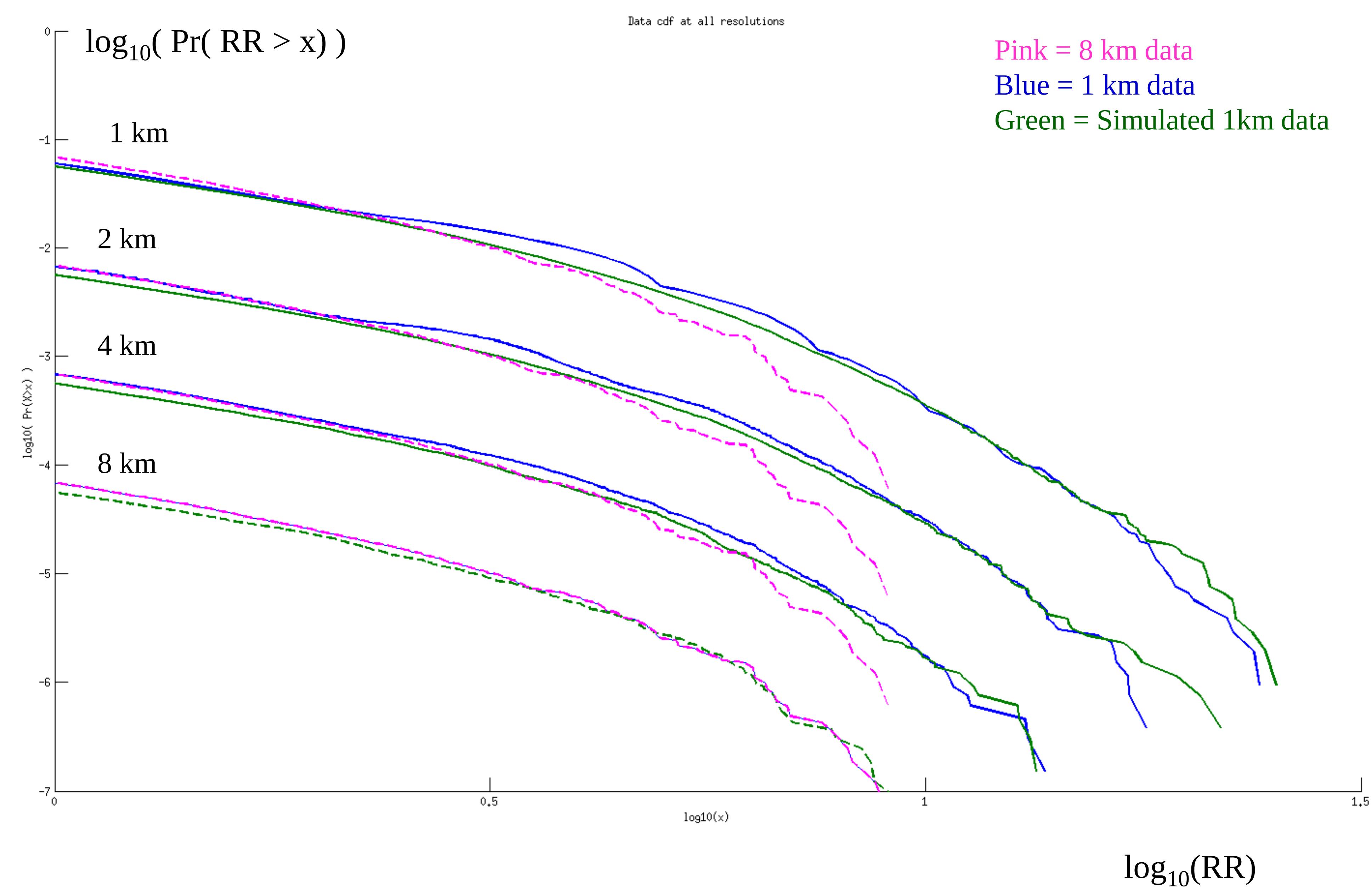


MeteoFrance Mosaics rain rate (26/03/2008)

Multifractal downscaling 32-8 km (example on 1 map)



Multifractal downscaling 8-1 km (example on 1 map)



Conclusion

- Calibration and validation procedures merging rainfall data of different resolutions are significantly scale-sensitive

- Reason: great variability characterized by multifractal statistics

- Multifractal formalism: tool to understand such artifacts
  - Z-R case: analytical solution
  - Other sensors: possibility to use multifractal simulators

- Multifractal downscaling = promising tool for rainfall data comparison and fusion

More info on Z-R scale-dependency

Theoretical and empirical scale dependency of Z-R relationships: Evidence, impacts, and correction

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Received 27 September 2012; revised 4 June 2013; accepted 4 June 2013.

[1] Estimation of rainfall intensities from radar measurements relies to a large extent on power-laws relationships between rain rates *R* and radar reflectivities *Z*, i.e., $Z = a^*R$^b. This relationship is generally applied unawarely of the scale, which is questionable since the nonlinearity of this relation could lead to undesirable discrepancies when combined with scale aggregation. Since the parameters (a,b) are expectedly related with drop size distribution (DSD) properties, they are often derived at disdrometer scale, not at radar scale, which could lead to errors at the latter. We propose to investigate the statistical behavior of *Z-R* relationships across scales both on theoretical and empirical sides. Theoretically, it is shown that claimed multifractal properties of rainfall processes could constrain the parameters (a,b) such that the exponent b would be scale independent but the prefactor a would be growing as a (slow) power law of time or space scale. In the empirical part (which may be read independently of theoretical considerations), high-resolution disdrometer (Dual-Beam

Journal of Geophysical Research 118, 14, 7435-7449, DOI: 10.1002/jgrd.50557

Multifractal formalism (recalls)

- Power-law spectra

$$E(k) \approx k^{-\beta}$$

β = 1-K(2) (< 1)

- Statistical moments = power-law of the resolution

$$\boxed{\left\langle \Phi_{\lambda}{}^{q} \right\rangle \approx \lambda^{K(q)}}$$

q =order of statistics

$K(q)$ = moment scaling function

λ = resolution (here, = 1/scale)

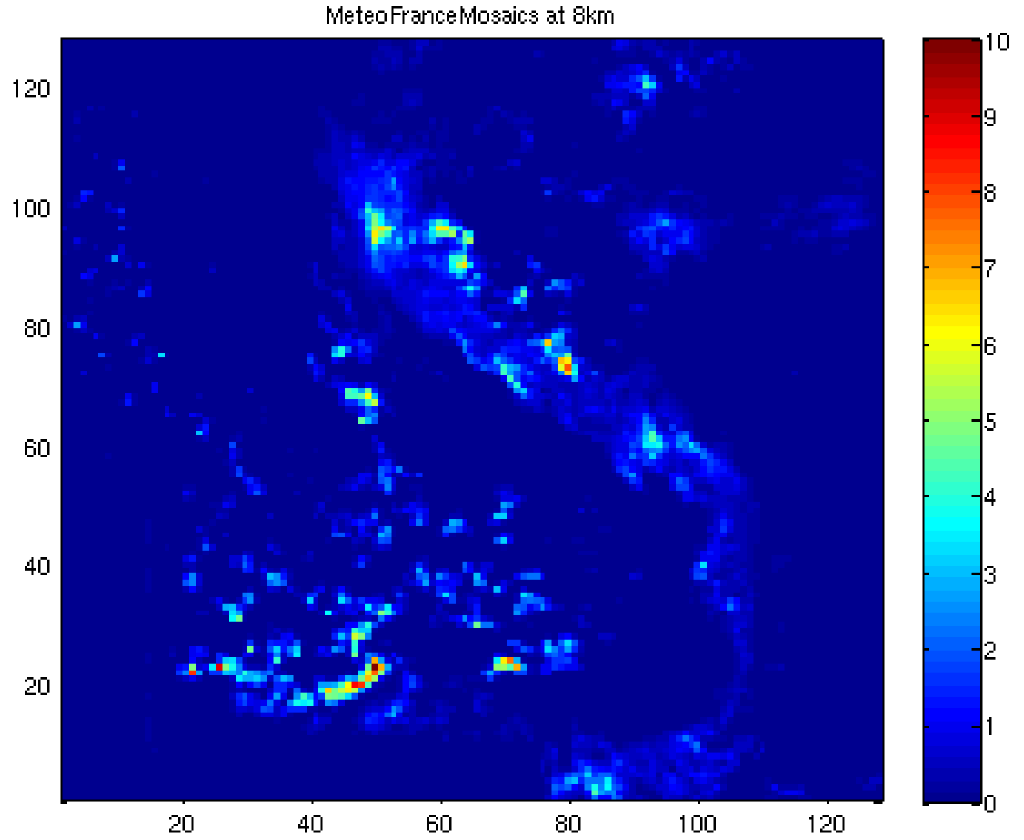
- Cascade phenomenology, with non – conservative variants (P(q) = K(q) – qH)

- Powers of multifractal fields are multifractal *(Tessier et al. 1993)*

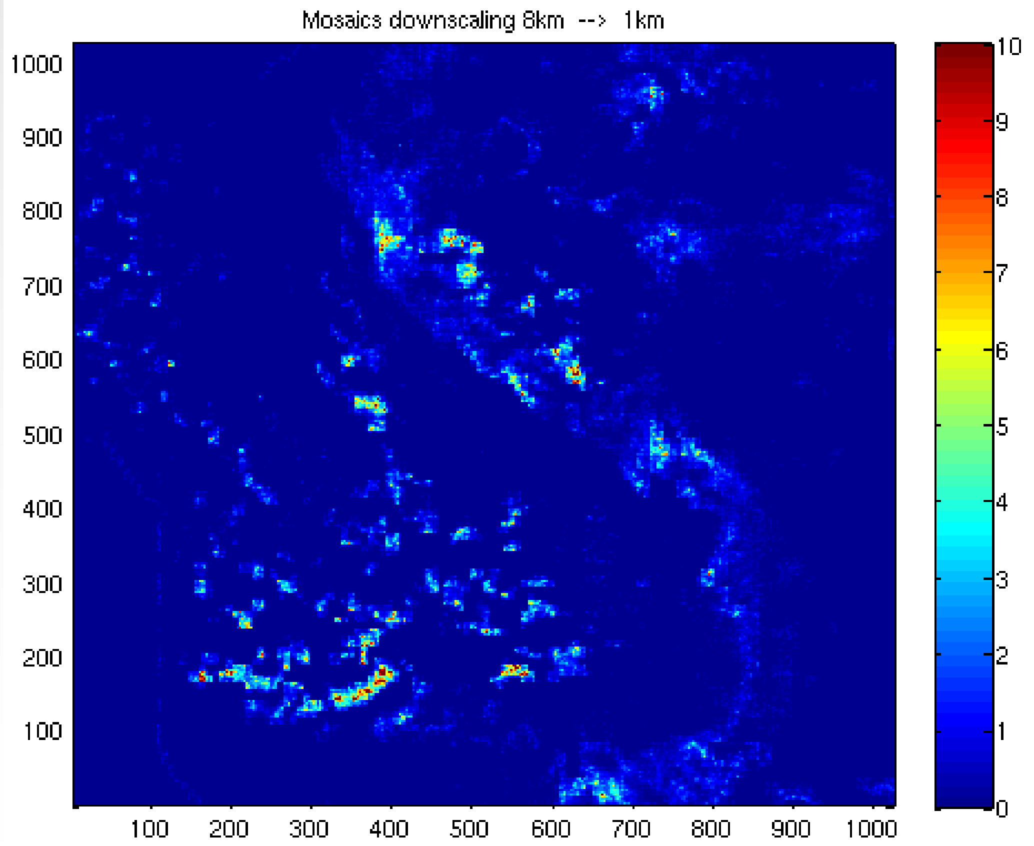
Inferring sub-pixel variability

- Possibility to simulate stochastic sub-pixel variability by extrapolating multifractal scaling laws…
- By construction, accurate retrievement of the CDF at multiple scales

Rainfall composite data aggregated at 8 km scale



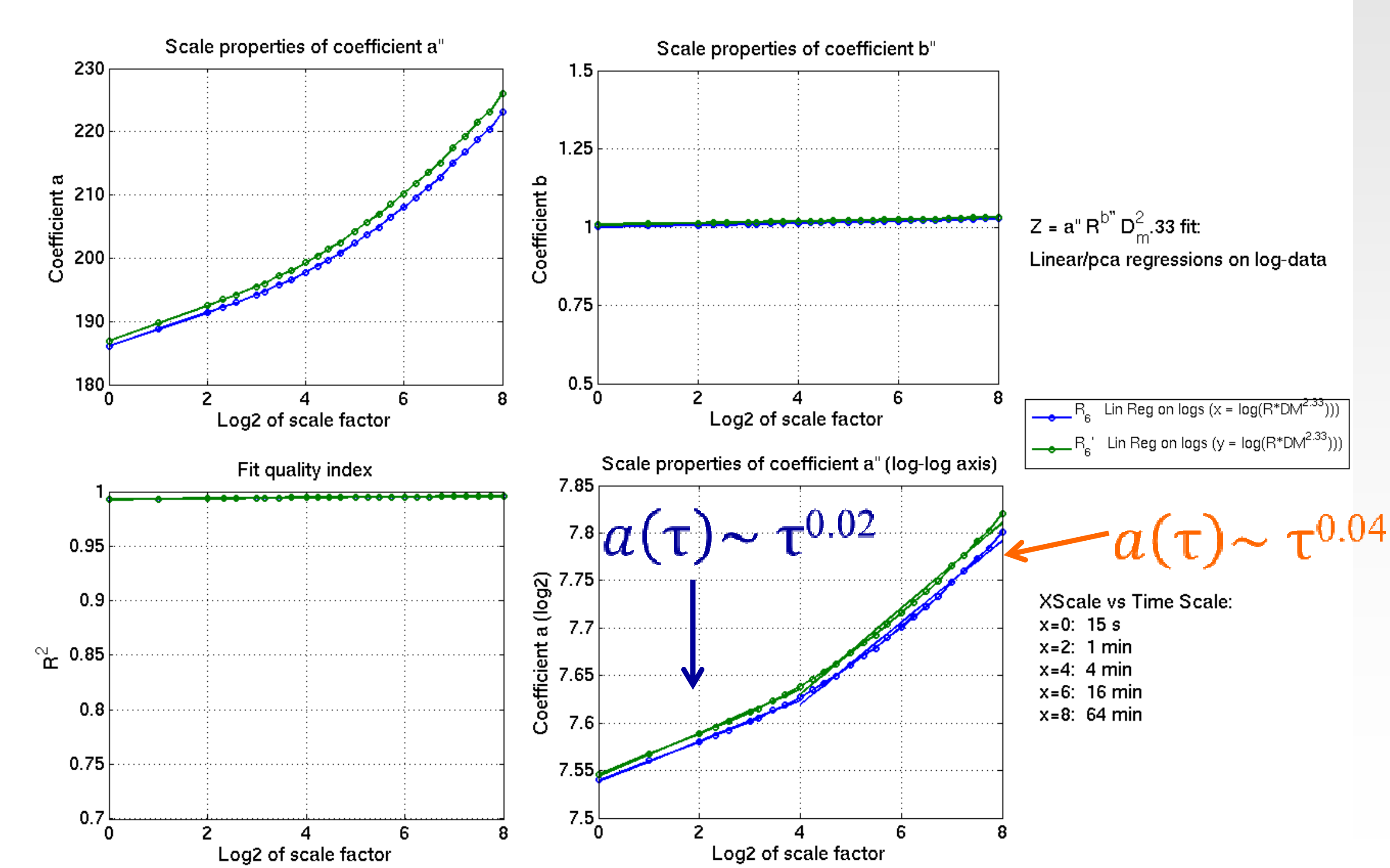
Rainfall composite data disaggregated at 1 km scale



MeteoFrance Mosaics rain rate (26/03/2008)

Scale behaviour of normalized Z-R laws

- (Multiscale) Normalized Z-R-N0*-Dm relationships are much more stable with scale with exponents 0.02 – 0.05



$$\left(\frac{Z}{N_0 *}\right) = a' \left(\frac{R}{N_0 *}\right)^{1.5}$$

$$Z = a'' R^{1.0} Dm^{2,33}$$

Scale behaviour of normalized Z-R laws

- Testud et al. (2001) defined DSD normalization parameters N0* and Dm (defined from 3rd and 4th DSD moments) such that N(D) = N0* f(D/Dm):

$$N_0^* = \frac{4^4}{6}\frac{M_3^5}{M_4^4} \qquad D_m = \frac{M_4}{M_3}$$

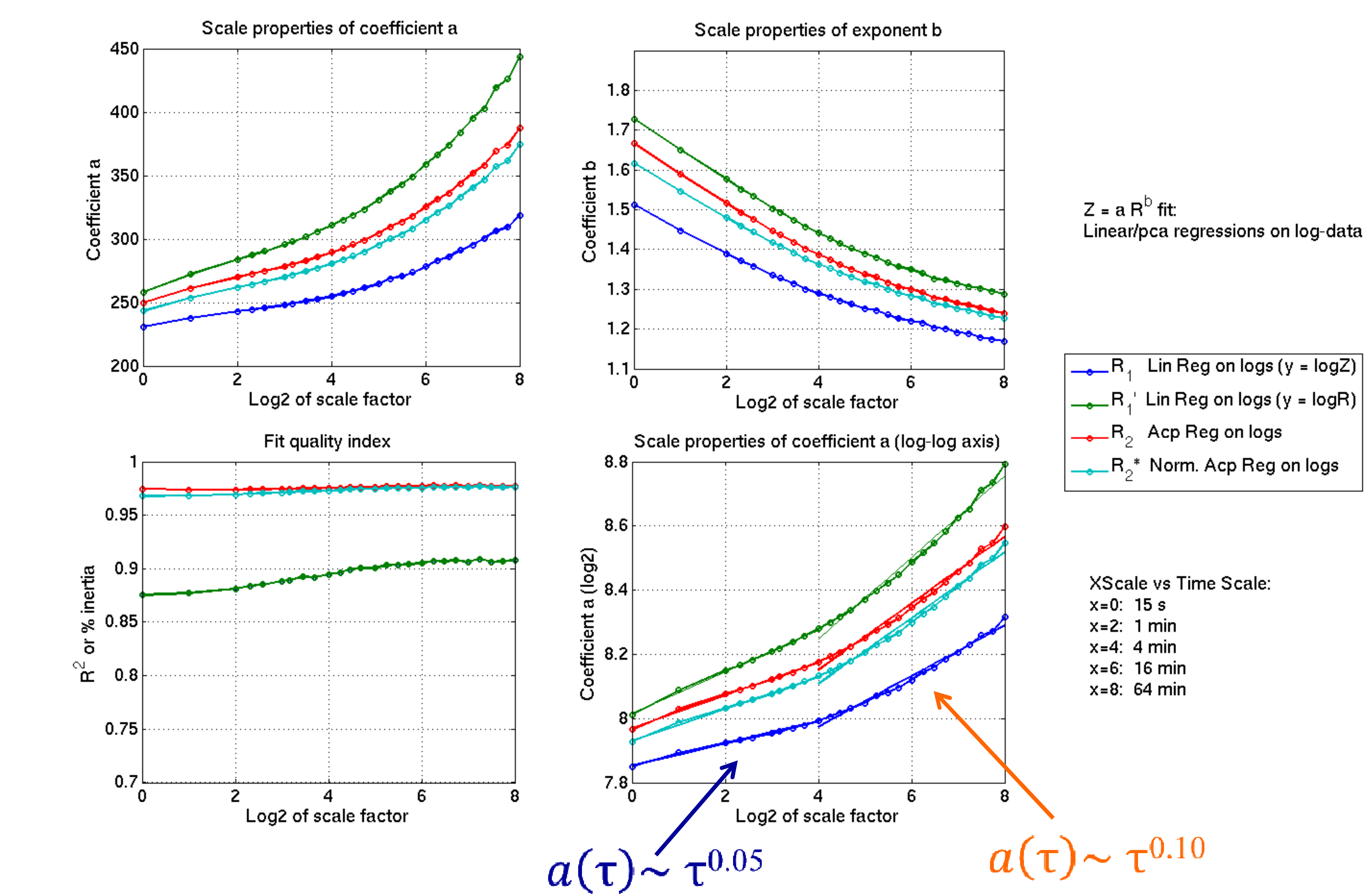
- Modified Z-R relationships

$$\boxed{\left(\frac{Z}{N_0*}\right) = a' \left(\frac{R}{N_0*}\right)^{1.5}} \qquad \boxed{Z = a'' R^{1.0} Dm^{2,33}}$$

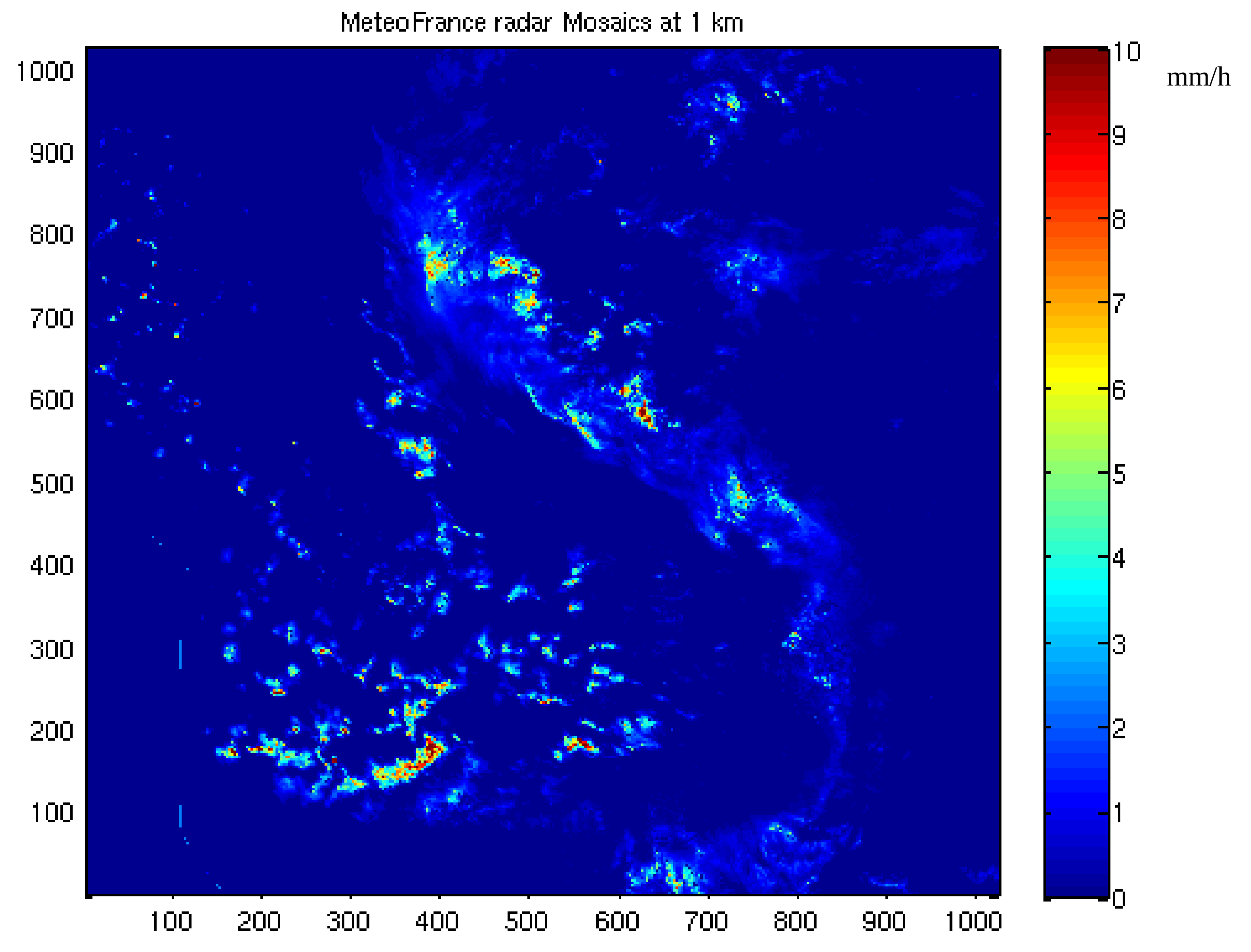
- Contrary to Testud et al., we estimated N0* and Dm at multiple scales from time aggregations of M3 and M4 of the DSD

- Then, same experimental methodology on the disdrometer data…

Scale-dependency of *a* and *b*



Rainfall is not smooth, but scaling!



Rainfall radar composite over France (26/03/2008)